Chapter 13

Formal methods

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13.1 Introduction

In mathematics a formal theory is one where conclusions can be drawn by looking at the *form* of expressions rather than by their *content*. This involves expressing the theory in a *formal language* (e.g. predicate logic) and perhaps even organizing it as a *formal* (or *axiomatic*) *system*. The term *formal* is also used to describe a theory which is made mathematically precise. The two notions are intimately related as mathematical precision often involves a degree of explicitness that does not leave content open to different interpretations.

Let us consider a trivial example. Suppose that we wish to express part of a theory of natural language syntax which says that sentences may consist of noun-phrases and verb-phrases. We might formulate our theory in natural language:

(1) A sentence may contain a noun-phrase and a verb-phrase.

However, we quickly notice that this leaves a number of things open to interpretation, if we formulate it this way. May it contain other things as well? Does it matter what order the noun-phrase and verb-phrase occur in? Of course, we can be more precise.

(2) A sentence may consist entirely of a noun-phrase followed by a verb-phrase.

And we could, of course, write other syntactic rules similarly.

(3) a. A noun-phrase may entirely consist of either a determiner followed by a common noun or a proper name.

b. A verb-phrase may entirely consist of either an intransitive verb or a transitive verb followed by a noun-phrase.

Writing a large grammar in this way with more complex rules would quickly become difficult to understand and we may never be sure that we had eliminated all sources of misinterpretation. Furthermore it would be quite complicated to express how to construct syntactic structures on the basis of these rules. It becomes much easier to define a *formal language* or *formalism* whose meaning we can make mathematically precise and which becomes much easier to read and to manipulate (e.g. when we want to draw conclusions about the syntactic structure associated with a particular string of words).

(4) a. $\text{S} \rightarrow \text{NP VP}$

b. $\text{NP} \rightarrow \left\{ \begin{array}{l} \text{Det N} \\ \text{PropN} \end{array} \right\}$

c. $\text{VP} \rightarrow \left\{ \begin{array}{l} \text{V}_i \\ \text{V}_t\ \text{NP} \end{array} \right\}$

We will explain some of this formalism below.

In this chapter we will first go through what it means to be formal in the first sense using logical inference as an example and show how formal logics are used in this connection (section 13.2). We shall then take a closer look at how formal systems are constructed (section 13.3) and in addition say something about how they are interpreted (section 13.4). Finally, we will give some typcial examples of how formal methods are used by linguists in syntax and semantics (sections 13.5.1 and 13.5.2).

13.2 Formal languages

An early example of formalization in order to facilitate correct inference is Aritotle's theory of syllogisms. Look at the following two inferences:

(5) All phonologists are linguists
Some linguists are semanticists
―――――――――――――――
∴ Some phonologists are semanticists

(6) Some phoneticians are linguists
All linguists are language experts
―――――――――――――――
∴ Some phoneticians are language experts

In (5) we have an example of an inference which is invalid, since the conclusion (that all phonologists are semanticists) can be false even if both of the premises (that all phonologists are linguists and some linguists are semanticists) are true. In (6) on the other hand we have a valid inference, since the conclusion must be true given that the premises are true.

Aristotle noted that the validity of an inference of this type (a *syllogism*) is independent of the specific meaning of the content words ('phonologist', 'linguist', 'semanticist', 'phonetician', 'language expert') which appear in the propositions and only depends on how the content words are determined by the function words such as 'all', 'some' and 'are'. This becomes even clearer if we substitute schematic symbols (or *variables*) for the content words in the above examples.

(7) All A are B
Some B are C
$\therefore$ Some A are C

(8) All A are B
Some B are C
$\therefore$ Some A are C

We can now say (with Aristotle) that *all* inferences which have the form of (7) – independently of which specific words stand in for A, B and C – are invalid, while *all* inferences which have the form (8) are valid. In this way we can now determine whether an inference is correct merely by considering its *form* – without taking account of the *content* of the terms which correspond to A, B and C. We have reduced a question of *content* (that is, whether a certain proposition follows logically from certain other propositions) to a question about *form* (that is, whether the proposition has a certain formal shape).

This should not be interpreted to mean that we are not interested in the content of the propositions. On the contrary, it is the content of our theoretical propositions which makes them interesting from a scientific point of view and which makes them be a theory *about* a certain phenomenon (for example, phonemes or parts of speech or speech acts). The idea is just that it is easier to guarantee correct inference if the rules for this inference can be formulated in terms of the form of the propositions rather than their content.

This, however, places certain constraints on the language in which we formulate our theories and this underlies a large part of the development of modern logic at the end of the nineteenth century and the beginning of the twentieth century. A red thread in this development was the idea of constructing an artificial language which was more suitable than natural language to the formulation of scientific theories and ensuring correct inference. The most well-known and widespread of these artificial languages is *predicate logic*, which was in all essentials constructed by the German logician and mathematician Gottlob Frege who is also known as the "father of modern logic". (A useful introduction to Frege's work can be found online in the *Stanford Encyclopedia of Philosophy*, `http://plato.stanford.edu/entries/frege`.)

One of the key characteristics of predicate logic (as well as other artificial languages constructed by logicians) is that it is *unambiguous*, that is, every expression of the language has exactly one interpretation. This is in contrast to natural languages which contain many ambiguous expressions. Consider the following sentence of Swedish:

(9) Pelle gillar Lisa
Pelle likes Lisa

This sentence can be interpreted in (at least) two ways. Firstly it can be interpreted so that 'Pelle' is the subject and 'Lisa' is the object, meaning the Pelle is the one who likes and Lisa is the object of his liking. But in Swedish the sentence can also be interpreted in a way such that 'Pelle' is the object and 'Lisa' is the subject so that Pelle is the object of Lisa's liking. (This interpretation corresponds to English 'It's Pelle that Lisa likes' or 'Pelle, Lisa likes'. In Swedish, like in many other Germanic languages other than English, the tensed verb always comes in second position.) In predicate logic these two meanings would be expressed with two distinct sentences (each unambiguous), which might look as follows:

(10) like(pelle,lisa)

(11) like(lisa,pelle)

In these formulae 'like' is a *predicate*, that is, an expression which is used to ascribe a property to an individual or (as in this case) to express a relation which holdes between individuals. The expressions 'pelle' and 'lisa' are called individual constants and are used to refer to particular individuals (exactly one individual per individual constant). In the examples above the individual constants are said to be *arguments* to the predicate, that is, they represent the individuals which stand in the relation in question. Note that the order between the arguments is significant, which means that (10) (unambiguously) says that Lisa is the object of Pelle's liking, whereas (11) says that Pelle is the object of Lisa's liking.[1]

Another very important (and intentional) property of the language of predicate logic is what is usually called the *logical form* of a sentence (those logical representation which is suitable for drawing correct inferences using inference rules) can be directly read off from its *syntactic form*. This is not something that is in general true for natural languages. Consider, for example, the following inferences:

(12) $\dfrac{\text{Pelle is nice}}{\therefore \text{ Someone is nice}}$

(13) $\dfrac{\text{Nobody is nice}}{\therefore \text{ Somebody is nice}}$

The premises in (12) and (13) can (roughly) be said to have the same syntactic form. They consist of a noun-phrase subject, followed by a copula verb and an adjectival complement. In spite of this the inference in (12) is valid whereas that in (13) is invalid. The reason for this is that the premises – in spite of their similar syntactic form – have different logical properties, that is, they have different *logical forms*. In predicate logic the corresponding formulae have quite different syntactic forms.

(14) $\dfrac{\text{nice(pelle)}}{\therefore \exists x \text{ nice}(x)}$

[1] A complete account of the language of predicate logic (including syntax and semantics) is beyond the scope of this presentation. We refer the reader to Allwood, Andersson and Dahl (1977).

(15) $\dfrac{\neg\exists x \text{ nice}(x)}{\therefore\ \exists x \text{ nice}(x)}$

The conclusion in (14) and (15) is a logical formula which can be read as: "there is at least one individual x such that x is nice" (that is, "there is someone who is nice"). The fact that this conclusion has completely different logical properties than the premise in (14) (for example, that it explicitly contradicts the conclusion) is apparent from the formula's syntactic form. Therefore predicate logic gives us, in addition to an unambiguous language, a language where *logical form* is directly related to *syntactic form* and where we can characterize correct inferences purely in terms of the form of formulae (just as Aristotle did for the special case of inferences which are called syllogisms). For example, we can say that any formula which has the same syntactic form as the premise in (14) logically implies the corresponding existential formula. We can express this as follows, where Φ stands for any predicate and α stands for any individual constant:

(16) $\dfrac{\Phi(\alpha)}{\therefore\ \exists x\ \Phi(x)}$

This is an example of a *logical rule of inference* (or *deduction rule*, that is, a rule which defines a valid inference in the language of predicate logic). This particular rule is usually called *existential generalization* (since one moves from a specific proposition about a particular indivdual to an general proposition about the existence of individuals with a particular property).

13.3 Axiomatzation and formal systems

Modern logic thus provides a formal language (actually many different languages) with unambiguous formulae whose logical form can be read directly of their syntactic form. This makes it possible to formulate logical rules of inference (for example, existential generalization) which refer solely to the syntactic form of formulae.

To formulate a theory (a set of theoretical propositions) in the formal language of logic leads therefore (in a sense) to increased clarity and precision. When one talks of formal theories one can mean something more than that the theory is formulated in predicate logic (or some other logical language). One can in addition require that the theory should be *axiomatized* and constitutes a *formal system*.

The idea of *axiomatization* goes back at least to the ancient Greeks where Euclid's geometry is usually cited as a typical example of an axiomatic theory (even though it was not formulated in predicate logic). The idea is that one sets up a number of basic true principles, or *axioms*, and then uses logical rules of inference to derive all true consequences, or *theorems*. We can illustrate the idea by formulating some simple axioms which could be included in a theory of Swedish or English phonology:

(17) (Axiom 1) All stops are consonants
(Axiom 2) No consonants are vowels
(Axiom 3) The phoneme /k/ is a stop

In predicate logic these axioms can be formulated as follows:

(18) (Axiom 1) $\forall x(\text{stop}(x) \rightarrow \text{consonant}(x))$
(Axiom 2) $\forall x(\text{consonant}(x) \rightarrow \neg\text{vowel}(x))$
(Axiom 3) stop(k)

The first axiom is to be read as: "for every object x it is the case that *if* x is a stop, *then* x is *not* a vowel".

Given that every stop is a consonant and that /k/ is a stop, then it follows that /k/ is also a consonant. In our axiomatic theory we can *prove* this by deriving the following sentence as a theorem from our axioms:

(19) (Theorem 1) consonant(k)

In order to be able to do this we must first equip ourselves two logical inference rules which we call Universal Instantiation (UI) and Modus Ponens (MP) respectively:

(20) a. (UI) $\dfrac{\forall x\Phi(x)}{\therefore\ \Phi(a)}$

b. (MP) $\dfrac{\begin{array}{l}(\Phi \rightarrow \Psi)\\ \Phi\end{array}}{\therefore\ \Psi}$

The first rule says that if we have a premise with a universally quantified formula, that is, a formula which begins with $\forall u$ (where u is an variable over individuals), then we may derive (that is, draw the conclusion) the formula which we obtain by removing $\forall u$ and substituting all occurrences of u in the remainder of the formula with an arbitrary individual constant a. (The expression $\Phi(x)$ is used to represent an arbitrary formula which contains one or more occurrences of the variable x, while $\Phi(a)$ represents the result of replacing all occurrences of x with the individual constant a.) With the help of the rule (UI) we can, for example, make the following inference in our phonological theory:

(21) $\dfrac{\forall x(\text{stop}(x) \rightarrow \text{consonant}(x))}{\therefore\ (\text{stop}(x) \rightarrow \text{consonant}(x))}$

This inference is valid since the consequence ("if /k/ is a stop then /k/ is a consonant") can be obtained from the premise by removing $\forall x$ and replacing all occurences of x in the remainder of the formula with the individual constant k (which denotes the phonem /k/).

The second inference rule (MP) says that if we have premises which consist of an implication (that is, an *if . . . then* formula) and the antecedent (*if*-clause) of the implication, then we can derive the consequent (*then*-clause) of the implication. This allows us, for example, to make the following inference in our phonological theory:

(22) $\dfrac{\begin{array}{l}(\text{stop}(k) \rightarrow \text{consonant}(k))\\ \text{stop}(k)\end{array}}{\therefore\ \text{consonant}(k)}$

The premises in (22) say that if /k/ is a stop then /k/ is a consonant. The conclusion is that /k/ is a consonant.

A *proof* in an axiomatic theory is a sequence of formulae $S_1, \ldots, S_n$, where the last formula S_n is the formula that is to be proved and where each preceding formula $(S_1, \ldots, S_{n-1})$ is *either* an axiom *or* can be derived from the preceding formulae by a logical inference rule. As an example we will now give a proof (in our axiomatic logical theory) that /k/ is a stop (that is, that the formula stop(k) is a theorem):

(23)	1.	$\forall x(\text{stop}(x) \rightarrow \text{consonant}(x))$	(Axiom)
	2.	$\text{stop}(k) \rightarrow \text{consonant}(k))$	(UI from 1)
	3.	$\text{stop}(k)$	(Axiom)
	4.	$\text{consonant}(k)$	(MP from 2, 3)

Finally we can give a definition of the concept *axiomatic theory*. In order to do this we first define a *formal system* (or *axiomatic system*) as a system consisting of the following three components:

1. A formal language which we can use to formulate theoretical propositions (for example, predicate logical with appropriate predicates and individual constants).
2. A set of axioms, that is formulae in the formal language
3. A set of inference rules, that is, rules that allow us to derive formulae in the formal language from other formulae in the formal language.

The *theory* which is defined by a formal system is the set of formulae which are either *axioms* of the system (that is, which are assumed to be basic truths) or are *theorems* (that is, which can be proved from the system's axioms by using the system's inference rules). As an example we can consider the (very simple and incomplete) phonological theory which is defined by Axioms 1–3 together with the inference rules UI and MP:

(24)	Axioms:	$\forall x(\text{stop}(x) \rightarrow \text{consonant}(x))$
		$\forall x(\text{consonant}(x) \rightarrow \neg\text{vowel}(x))$
		$\text{stop}(k)$
	Theorems:	$(\text{stop}(k) \rightarrow \text{consonant}(k))$
		$\text{consonant}(k)$
		$(\text{consonant}(k) \rightarrow \neg\text{vowel}(k))$
		$\neg\text{vowel}(k)$

13.4 The interpretation of formal systems: model theory

We have considered the fact that the aim of constructing formal systems is to reduce content to form to facilitate the construction of correct proofs because certain things follow from our theoretical claims. A formal system is only interesting (for a linguist) if its axioms can be interpreted as true theoretical claims about a certain (linguistic) domain, and if the theorems which can be derived can be interpreted as true propositions about this domain.

A formal system where all of the theorems which can be derived (proved) in the system are also true propositions (given an intended interpretation of the axioms of the system) is said to be *sound*. Soundness therefore means that you cannot prove something which is false. In *quasi*-logical notation:

Soundness = (Provability → Truth)

Ideally one would also desire that the opposite holds, that is, that all true theorems can be derived (proved) in the system. In this case, we say that the system is *complete*:

Completeness = (Truth → Provability)

A system which is not sound constitutes an incorrect theory, since it is possible to prove propositions that are not true. Therefore, when one talks about whether a system is complete or not, one normally assumes that the system is sound. One of the most important driving forces behind the development of formal systems was the idea that one would be able to construct a complete system for mathematics, that is, a formal system where one would be able to mechanically prove all the true propositions of mathematics.[2] It came therefore as a great surprise for most people when the Austrian logician Kurt Gödel showed in the 1930's that it is impossible to give a complete axiomatization even of simple arithmetic (addition and multiplication with the natural numbers). In other words there will always be true mathematical propositions which cannot be proved in the formal system.

Gödel's result did not, however, show that formal systems are uninteresting. Even an incomplete system is a correct theory as long as it is sound, though it will only be a partial theory. Furthermore there are many other areas where it is possible to construct complete axiomatic theories. But an important consequence of Gödel's result was that people also got interested in the semantic aspect of formal systems and tried to give a more exact characterization of what is involved in giving an *interpretation* for a formal system and what it means for a formul to be true or false (under a given interpretation of the system). The branch of logic where one studies these questions is now called *model theory* because the notion of *model* plays a central role in this connection.

A classical *model* (or an *interpretation*) for a formal system like predicate logic consists of two parts:

1. A set of objects A, called the model's *domain* or *universe* (that is, the set of objects which one can talk about in the system).
2. An interpretation of the system's predicates and individual constants, where
   (a) each individual constant is assigned (exactly) one object in the domain of the model (that is, in the set A)
   (b) each predicate which takes *one* argument is assigned a set of objects in the domain of the model (that set of objects of which the predicate can truthfully be predicated)
   (c) each predicate which takes *two* arguments is assigned a set of *pairs* of objects in the domain of the model (that set of pairs of which the predicated can truthfully be predicated)
   ... (and so on for predicates which take more that two arguments)

[2] Many people also imagined that such a system could be eventually extended to include the whole of science.

As an example we can give a model for the simple phonological system which we sketched in the previous section:

1. The domain of the model is $A = \{x|x \text{ is a phoneme of Swedish}\}$. (This set contains among other things the phoneme /k/ which we used in our examples earlier.)
2. The model's interpretation of predicates and individual constants is as follows (where the notation $[\![a]\!]$ represents the interpretation of a):
   (a) $[\![k]\!] = /\mathrm{k}/$
   (b) $[\![\text{stop}]\!] = \{x|x \in A \text{ and } x \text{ is a stop}\}$
   $[\![\text{consonant}]\!] = \{x|x \in A \text{ and } x \text{ is a consonant}\}$
   $[\![\text{vowel}]\!] = \{x|x \in A \text{ and } x \text{ is a vowel}\}$

Using the notion of model we can now define what it means for formulae in a formal system (or in predicate logic in particular) to be true or false relative to a particular model (or interpretation). Giving a complete truth-definition for the language of predicate logic would go beyond the scope of the present work,[3] but we can illustrate the technique for some simple kinds of expressions:

1. A formula of the form $P(a)$, where P is a predicate and a an individual constant is true in a model M if and only if $[\![a]\!] \in [\![P]\!]$ (that is, just in case the object that a stands for is in the set of objects which P stands for according to the model M); otherwise $P(a)$ is false in M.
2. A formula of the form $\neg\Phi$, where Φ is an arbitrary formula, is true in a model M if and only if Φ is false in M; otherwise $\neg\Phi$ is false in M.

Using these definitions we can see that the following formulae are true according to the definition characterized above for our phonological system:

(25) stop(k)
consonant(k)
$\neg$vowel(k)

On the other hand we can show that the following formulae are false:

(26) vowel(k)
$\neg$stop(k)
$\neg$consonant(k)

Finally we can use model theoretic semantics for formal languages and formal systems to give a characterization of what it means for a theory T defined by a formal system S to be sound and complete with respect to a class of models $\mathfrak{M}$:

T is sound	$\Leftrightarrow$	Each formula which is provable in S is true in all models in $\mathfrak{M}$ where the axioms of S are true.
T is complete	$\Leftrightarrow$	Each formula which is true in all models in $\mathfrak{M}$ where the axioms of S are true is provable in S.

[3] As before the reader is referred to Allwood, Andersson and Dahl (1977)

13.5 Formal methods in linguistics

Formal methods using formal languages and formal systems of the kind we have discussed above can be applied in any scientific field and are not limited to any particular part of linguistics. One can create formal theories in sociolinguistics, psycholinguistics, discourse analysis and so on. The tradition has been in linguistics, however, that formal methods are most extensively applied in *syntax* and *semantics*. We shall therefore in conclusion look a little more closely at the formal methods employed in "formal syntax" and "formal semantics".

13.5.1 Formal syntax

In syntactic research since the 1950's (at least) a particular kind of formal system known as *formal grammars* has been employed. This kind of syntactic theory build on two important assumptions:

1. a language - either natural or artificial - can be seen as a (normally infinite) set of *strings of symbols*, where a string of symbols is a (finite) string of symbols taken from a (finite) vocabulary.

2. a syntactic description of a language is to (minimally) define the set of strings which make up the language

As an example we can consider that the strings which appear in (27a) are included in English (that is, they constitute grammatically well-formed expressions of English), while those that appear in (27b) are not included in English in spite of the fact that all the symbols (words) are English.[4]

(27) a. the little dog sleeps
the dog chases the cat

b. *little the sleeps dog
*cat the dog the chases

A formal grammar consists of four components:

1. A (finite) set of *terminal symbols* (symbols which can be included in the strings of the language).

2. A (finite) set of *non-terminal symbols* (category symbols or auxiliary symbols which are used in the derivation of the strings of the language).

3. A *start symbol* (taken from to stock of non-terminal symbols).

4. A (finite) set of *rewrite rules* (rules which allow us to replace on strings of symbols with another in a derivation).

[4]Placing '*' before a string represents that it is not well-formed (i.e. that it is not included) in the language under consideration.

As an example there follows a (very simple) formal grammar for a small fragment of English:

1.	Terminal symbols:	{the, little, dog, cat, sleeps, chases}
2.	Non-terminal symbols:	{S, NP, VP, DET, A, N, V}
3.	Start symbol:	S
4.	Rewrite rules:	{S $\rightarrow$ NP VP, NP $\rightarrow$ DET A N, NP $\rightarrow$ DET N, VP $\rightarrow$ V NP, VP $\rightarrow$ V}

Given this grammar we can now derive *terminal strings* (that is, strings which consist solely of terminal symbols) from the start symbol S by applying the grammar's rewrite rules. Examples of such derivations are given in (28).

(28) a.
1. S
2. NP VP
3. DET A N VP
4. DET A N V
5. the A N V
6. the little N V
7. the little dog V
8. the little dog sleeps

b.
1. S
2. NP VP
3. DET N VP
4. DET N V NP
5. DET N V DET N
6. the N V DET N
7. the dog V DET N
8. the dog chases DET N
9. the dog chases the N
10. the dog chases the cat

The derivations in (28) constitute a "proof" that the strings *the little dog sleeps* and *the dog chases the cat* are included in the language that is defined (or *generated*) by the grammar we specified above. (Since these strings are also included in English, one could see the above grammar as a first step towards a formal syntactic description of English.)

Formal grammars can be seen as a kind of formal system, but they differ from the formal systems we discussed earlier in that the objects that are derived ("proved") within the system (the "theorems" of the system) are not true theoretical propositions concerning that part of reality we want to describe but well-formed strings of the language which we wish to describe the syntax of.

The study of formal grammars was initiated by Noam Chomsky and others during the 1950's and has basically continued in its purest form within the discipline known as mathematical linguistics which lies at the intersection of linguistics, logic and computer science.[5] Within modern syntactic theory various further developments and alternatives to the simple type of grammar illustrated above are used. Among these are for example feature-based grammars which use unification. This

[5] The interested reader is referred to Partee, ter Meulen and Wall (1990).

is not the place to describe these but excellent introductions to early versions of such grammars can be found in Shieber (1986) and Kay (1992).

13.5.2 Formal semantics

The theoretical approach that usually goes under the name “formal semantics” in the literature is actually not an example of formal systems in the strict mathematical sense in that it usually does not use formal systems in the sense described above. It represents an attempt to characterize the semantics of natural languages using the methods developed for the semantic characterization of formal languages and therefore *model theoretic semantics* (for natural languages) is an appropriate term to describe much of the work in formal semantics.

Model theoretic semantics involves defining truth conditions for sentences in the language under study (with respect to a model or interpretation). Such truth conditions might, for example, look like the following:

(29) A sentence of the form NP VP, where NP is a referential noun-phrase and VP is a (finite) verb-phrase is true in a model M if and only $[\![\text{NP}]\!] \in [\![\text{VP}]\!]$ (that is, if the referent of the noun-phrase is included in the extension of the verb-phrase in M).

For example, the sentence ‘Pelle is a linguist’ is true in a model M if and only if the individual which ‘Pelle´ refers to in M is included in the extension of the noun ‘linguist´ (that is, the set of linguists) in M.

Often the truth conditions for natural language sentence are given indirectly by first translating them to a formal language. This can, for example, involve translating ‘Pelle is a linguist´ to ‘linguist(pelle)´ and then saying that the truth conditions of the logical formula are the truth-conditions of the English sentence. The reader who wishes to read more about this kind of syntax can find an introduction in Cann (1993).

References

Allwood, Jens, Lars-Gunnar Andersson and Östen Dahl (1977) *Logic in Linguistics*, Cambridge: Cambridge University Press.

Cann, R. (1993) *Formal Semantics*, Cambridge, Cambridge University Press.

Kay, M. (1992) Unification, in *Computational Linguistics and Formal Semantics,*, ed. by M. Rosner and R. Johnson, Cambridge: Cambridge University Press.

Partee, B.H., A. ter Meulen and R.E. Wall (1990) *Mathematical Methods in Linguistics*, Springer.

Shieber, S. (1986) *An Introduction to Unification-base Approaches to Grammar,* Stanford: CSLI Lecture Notes.

Study questions

1. What are formal methods according to the authors? What would be the advantages of a formal theory over an informal one? What could be the disadvantages?

2. What is a formal system? Explain and give examples.

3. What is model theory? Explain how one can use model theory to characterize the concepts of soundness and completeness of formal systems.

4. (relating to Kay) Construct a unification based grammar for the following fragment of (standard) English:

   he snores
   *he snore
   they snore
   *they snores
   he can help them
   *him can help they
   they can help him
   *them can help he